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Electroweak Interactions at an Infinite Sublayer Quark Level

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In a previous paper, we proposed the infinite sublayer quark model, in which there exists an infinite number of quarks u_∞ and antiquarks u_∞^C at an infinite sublayer level. By applying the standard model of the electroweak interactions to the weak isospin doublet $(u_\infty, u_\infty^C)^T$, it is shown that there exists only one gauge field W_μ^3, from which the electromagnetic field $A_\mu = W_\mu^3 \cos\theta_W$ and the neutral vector boson field $Z_\mu^0 = W_\mu^3 \sin\theta_W$ are derived.

1. INTRODUCTION

Recently the quark model has been widely believed and established in modern particle physics. For example, the proton (p) and the neutron (n) are made of u and d quarks, such as $p = uud$ and $n = udd$. The sublayer quark model has been proposed by some authors (Pati and Salam, 1974; Terazawa *et al.*, 1977; Glashow, 1977; Ne'eman, 1979; 't Hooft, 1979; Harari, 1979; Shupe, 1979). By analyzing deep inelastic electron–proton scattering, Kogut and Susskind (1974*a,b*) considered the existence of a further sublayer parton. Based on this assumption, we proposed the infinite sublayer quark model (Sekine, 1985). This model implied that the proton (p) and the neutron (n) are made of u_1 and d_1 quarks, so that $p = u_1u_1d_1$ and $n = u_1d_1d_1$. Furthermore, u_1 and d_1 quarks are made of u_2 and d_2, etc. In summary, u_N and d_N quarks at level N are made of u_{N+1} and d_{N+1} quarks at level $N+1$, such as $u_N = (u_{N+1}, u_{N+1}, d_{N+1})$ and $d_N = (u_{N+1}, d_{N+1}, d_{N+1})$ where

$$N = 1, 2, 3, \ldots, \infty$$

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Here, the u_N and d_N quarks have quantum numbers of spin $J=1/2$, baryon number $B=1/3^N$, isospin $I=1/2$, third component of isospin $I_3=\pm 1/2$, and fractional electric charge $Q=[(1\pm 3^N)/(2\times 3^N)]|e|$, where $|e|$ is the electron charge. Thus, at $N=\infty$, an infinite number of pointlike quarks (u_∞) and antiquarks ($u_\infty^C=d_\infty$) is considered as constituting the nucleon. The superscript C means charge conjugation. The ultimate particle u_∞ has quantum numbers of $J=1/2$, $I=1/2$, $I_3=1/2$, and $Q=1/2|e|$. Thus, all quantum numbers of the u_∞ quark are just one-half and this fermion will behave as if it was a lepton, since the baryon number approaches zero at an infinite sublayer level.

Glashow, Weinberg, and Salam proposed the electroweak model of $SU(2)_L\times U(1)$, which is now widely accepted as the standard model in modern particle physics (Glashow, 1961; Weinberg, 1967; Salam, 1968).

In the following, we shall apply this $SU(2)_L\times U(1)$ model to the interactions at the infinite sublayer quark level.

2. ELECTROWEAK INTERACTIONS AT THE INFINITE SUBLAYER QUARK MODEL

Consider the quantum numbers of weak isospin $t_3=1/2$ and $t_3=-1/2$ for the ultimate particles u_∞ and u_∞^C, respectively. The hypercharge of u_∞ and u_∞^C quarks becomes zero by applying the Nishizima–Gell-Mann relation to weak quantum numbers. Furthermore, we assume that u_∞ and u_∞^C quarks are massless.

Thus, the weak isospin doublet is written as $(u_\infty, u_\infty^C)^T$ under $SU(2)$. The superscript T means "transposed."

From these, the Lagrangian of the kinetic energy term and the electroweak interactions at the infinite sublayer level is written as

$$L=\overline{\chi_L}\gamma^\mu(i\partial_\mu-(g/2)\boldsymbol{\tau}\cdot\mathbf{W}_\mu)\chi_L \tag{1}$$

where $\chi_L=(u_\infty, u_\infty^C)_L^T$, $u_{\infty L}^C$ means charge conjugation operation of $u_{\infty L}$, the subscript L means the left-handed particle, g is the coupling constant, $\mathbf{W}_\mu$ are three gauge fields of $SU(2)_L$, and $\boldsymbol{\tau}/2$ is a generator of $SU(2)_L$.

First, we consider the free term of the Lagrangian in equation (1):

$$\begin{aligned} L_{\text{free}} &= \overline{\chi_L}\gamma^\mu i\,\partial_\mu\chi_L \\ &= \overline{(u_\infty)_L}i\gamma^\mu\,\partial_\mu(u_\infty)_L+\overline{(u_\infty^C)_L}i\gamma^\mu\,\partial_\mu(u_\infty^C)_L \\ &= \overline{(u_\infty)_L}i\gamma^\mu\,\partial_\mu(u_\infty)_L-[\partial_\mu\overline{(u_\infty)_L}]i\gamma^\mu(u_\infty)_L \end{aligned} \tag{2}$$

From the Euler–Lagrange equation, we obtain the Dirac equation for the massless particle,

$$i\gamma^{\mu}\,\partial_{\mu}(u_{\infty})_L=0 \tag{3}$$

If we consider a right-handed particle singlet, this equation is independent of the left- and right-handed particles, as in the framework of the standard model of $SU(2)_L\times U(1)$.

Next, we will find the conditions that the Lagrangian in equation (1) is invariant under $SU(2)_L\times U(1)$. For simplicity, we consider the infinitesimal $SU(2)_L\times U(1)$ symmetry without losing generality. As seen in equation (1), the term of $U(1)$ does not appear explicitly, since the hypercharge is zero.

Equation (1) is invariant under the following gauge transformation:

$$\mathbf{W}'_{\mu}=\mathbf{W}_{\mu}-\partial_{\mu}\boldsymbol{\alpha}-g\boldsymbol{\alpha}\times\mathbf{W}_{\mu} \tag{4}$$

$$\chi'_L=[1+i(g/2)\boldsymbol{\alpha}\cdot\boldsymbol{\tau}]\chi_L \tag{5}$$

where $\boldsymbol{\alpha}$ are parameters in $SU(2)_L$.

However, u_{∞} and u_{∞}^C quarks are not independent of each other. Therefore, certain conditions should be added to equation (5). These conditions are derived as follows:

First, we rewrite equation (5) using the isospinor components as

$$(u_{\infty})'_L=[1+i(g/2)\alpha^3](u_{\infty})_L+i(g/2)(\alpha^1-i\alpha^2)C\gamma^0(u_{\infty})^*_L \tag{6}$$

$$(u_{\infty}^C)'_L=i(g/2)(\alpha^1+i\alpha^2)(u_{\infty})_L+[1-i(g/2)\alpha^3]C\gamma^0(u_{\infty})^*_L \tag{7}$$

where C means charge conjugation operator.

From equation (6), we obtain

$$\begin{aligned}(u_{\infty}^C)'_L\equiv(u'_{\infty})^C_L=C\gamma^0(u'_{\infty})^*_L\\ =-i(g/2)C\gamma^0(\alpha^1+i\alpha^2)C\gamma^0(u_{\infty})_L\\ +C\gamma^0[1-i(g/2)\alpha^3](u_{\infty})^*_L\end{aligned} \tag{8}$$

Equation (7) is equivalent to equation (8) under the following conditions:

$$\begin{aligned}C\alpha^1&=-\alpha^1C\\ C\alpha^2&=-\alpha^2C\\ C\alpha^3&=\alpha^3C\end{aligned} \tag{9}$$

From equation (9), it is easily seen that $\alpha^1=\alpha^2=0$. In this case, W_{α}^1 and W_{μ}^2 do not exist and only one gauge field W_{μ}^3 does exist, since we construct the doublet $(u_{\infty},u_{\infty}^C)^T_L$, and $u_{\infty L}$ and $(u_{\infty}^C)_L$ quarks are not independent of each other. From this, we obtain the electromagnetic field

$A_\mu = W^3_\mu \cos\theta_W$ and the neutral vector boson field $Z^0_\mu = W^3_\mu \sin\theta_W$, where θ_W is the Weinberg angle.

3. CONCLUSION

We examined the electroweak interactions of $SU(2)_L \times U(1)$ at the infinite sublayer quark level and concluded that there exists only one gauge field W^3_μ associated with the electromagnetic field and the neutral vector boson field.

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